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DEVELOPMENT OF CROSS AXIS ENGINE DYNAMIC MODEL, DSV-4B

APRIL 1964
DOUGLAS REPORT SM-46535

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PREPARED FOR:
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION
CONTRACT NO. NAS7-101

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PREFACE

The purpose of this report is to develop a simplified dynamic model of the S-IVB thrust structure and engine for use in control system analysis. This model contains a sufficiently accurate representation of the structure to allow evaluation of the control system response due to the effect of structural flexibility.

This analysis was performed in compliance with the request made at the September 19, 1963, splinter meeting of the S-IVB Dynamics and Control Working Group, held at the Marshall Space Flight Center. The report is transmitted to partially fulfill the requirements of Contract Number NAS 7-101 as noted in Douglas Aircraft Company Report SM-41410: Data Submittal Document Saturn S-IVB System, Item 3.8, dated March 1962.

ABSTRACT

Five deflection influence coefficients for the thrust structure were converted to engine coordinates and simplified to a four degree of freedom system. The lumped compliance of the actuators and engine were then calculated and added to the system. These influence coefficients along with the mass characteristics of the engine were then used to determine the modal frequencies and mode shapes of the four degree of freedom system.

The system was then reduced to two degrees of freedom. Rotational deflection in the pitch and yaw planes was used. The mass characteristics of the engine were used again and the vibrational modes were found. Frequencies of the two modes determined by this analysis were found to be within 9% of the two dominate modes of the first analysis.

Descriptors

S-IVB
Thrust Structure
Dynamic Analysis
Influence Coefficients
Spring Rates
Matrix Algebra

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LIST OF SYMBOLS

h - translational deflection (in)

ϕ - rotational deflection of the engine (rad)

P - applied load (lbs)

M - moment applied to the engine (in lbs)

l - radial distance from engine centerline to actuator attach point (in)

ξ - generalized coordinate

K - spring constant (lbs/in)

$\left[\bar{M}\right]$ - mass matrix

$\left[\phi\right]$ - influence coefficient matrix

$\left[T\right]$ - transformation matrix

Subscripts

i - direction in thrust structure coordinates (1,2,3,4,5)

θ - pitch plane in engine coordinates

ψ - yaw plane in engine coordinates

a - actuator

S - structure

E - engine

A - lumped engine and actuator spring terms

1. INTRODUCTION

The inertial and elastic forces as sensed by the S-IVB engine actuator must be adequately predicted for inclusion in the engine servo design. Thus, a dynamic model including engine dynamics and thrust structure compliances must be formed.

The engine dynamics are presented in Reference 2 as a simple spring at the actuator attach point connected to a rigid engine. The inertial properties are the rigid body engine moment of inertia and mass. This model has been incorporated into the servo analysis for some time. No attempt will be made to alter the inertial formulation within this report.

The thrust structure compliance is the unknown that must be developed into a form acceptable to servo analysis techniques. The thrust structure engine attach point compliances were described by a set of influence coefficients in Reference 1. Matrix algebra methods[1] were then employed in this report to transform the influence coefficients from the thrust structure coordinates to an engine system of coordinates.

The equations of motion were formed for the engine coordinates, and the equations were solved for frequencies and mode shapes. The system was then reduced to two degrees of freedom using rotation in the pitch and yaw planes only. An analysis was again performed and the modal frequencies and mode shapes were determined. The frequencies of the two modes determined by this analysis were found to be very near the two dominate modes of the first analysis. This close proximity showed the system could be reduced to two degrees of freedom and still retain a desirable accuracy. An effective spring constant for the system was determined from this reduced matrix.

A mechanical analog for the two degrees of freedom system was then devised. This dynamic model possesses the same properties as the reduced system and is adequate for performing control system design.

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[1] Matrix algebra methods were used throughout this report to perform transformations and list simultaneous equations. A good reference for general matrix methods is Frazer, Duncan and Collar, Elementary Matrices, Cambridge Press, 1960.

2. CONVERSION OF THRUST STRUCTURE INFLUENCE COEFFICIENTS TO ENGINE COORDINATES

2.1 Thrust Structure Influence Coefficients

The influence coefficient matrix equation describing the S-IVB thrust structure (Reference 1) is:

$$\begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{Bmatrix} = (10^{-9}) \begin{bmatrix} 855 & -10 & -10 & 706 & 706 \\ -10 & 4844 & -40 & -1398 & 64 \\ -10 & -40 & 4844 & 64 & -1398 \\ 706 & -1398 & 64 & 3108 & -25 \\ 706 & 64 & -1398 & -25 & 3108 \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix} \qquad (1)$$

Or, in abbreviated form:

$$\begin{Bmatrix} h_i \end{Bmatrix} = \begin{Bmatrix} \phi_s \end{Bmatrix} \begin{Bmatrix} P_i \end{Bmatrix} \qquad (1)$$

The coordinate system used, h_1 through h_5, is presented graphically in Figure 1. Positive forces and deflections are in the direction shown by the arrows.

2.2 Transformation of Influence Coefficients to Engine Coordinates

For control system analysis, the engine rotation is the most convenient coordinate to use. A second coordinate system was therefore devised to describe engine movement directly (Figure 2). The influence coefficients in the new system will define translation of the gimbal point and rotation of the engine centerline in response to forces and movements in these coordinates. This new coordinate system will henceforth be referred to as the "engine coordinate system."

2.2.1 Coordinate Transformation

The matrix equation given in equation 2 describes deflection in the engine coordinate system in terms of deflection in the thrust structure coordinate system.

THRUST STRUCTURE COORDINATE SYSTEM

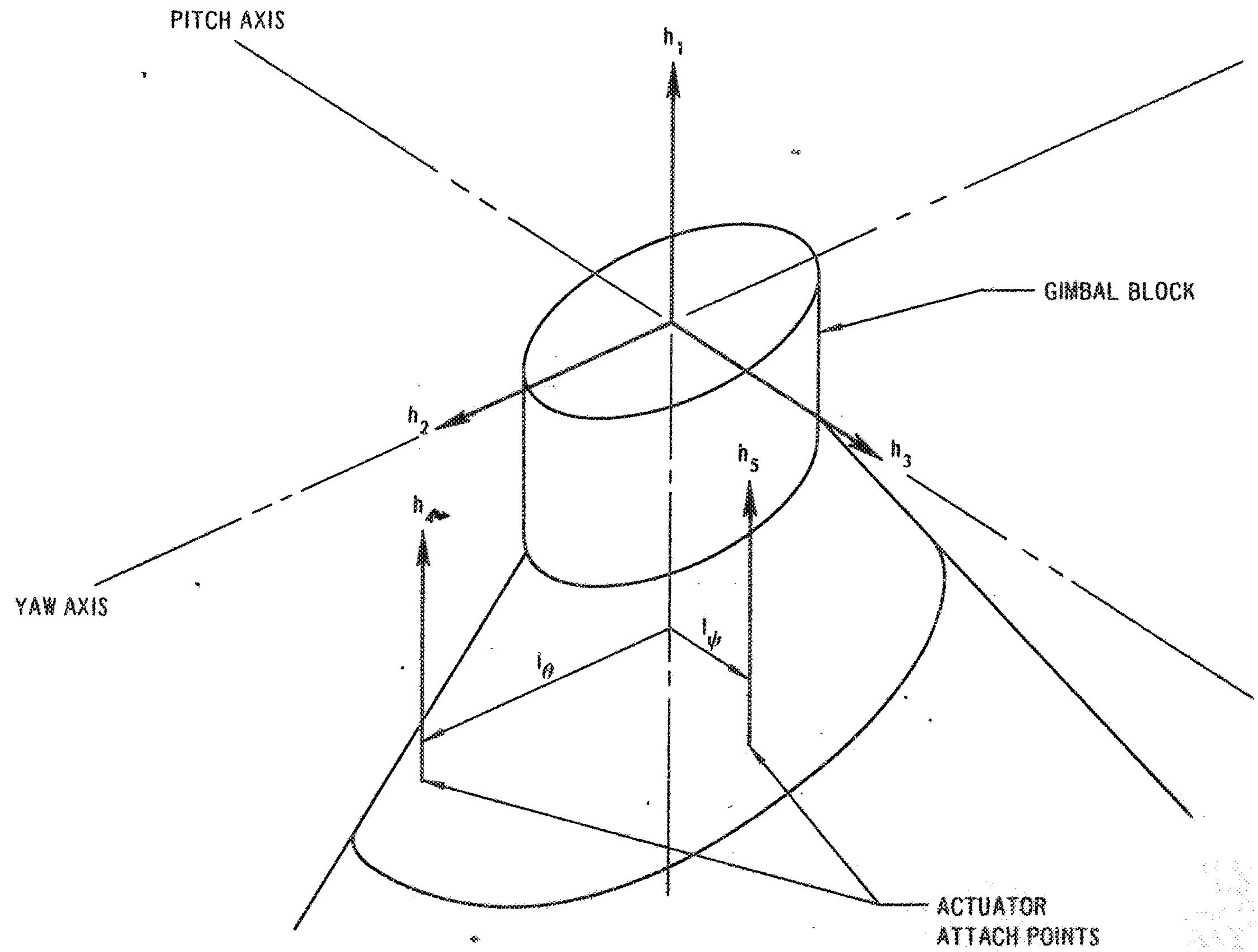


FIGURE 1

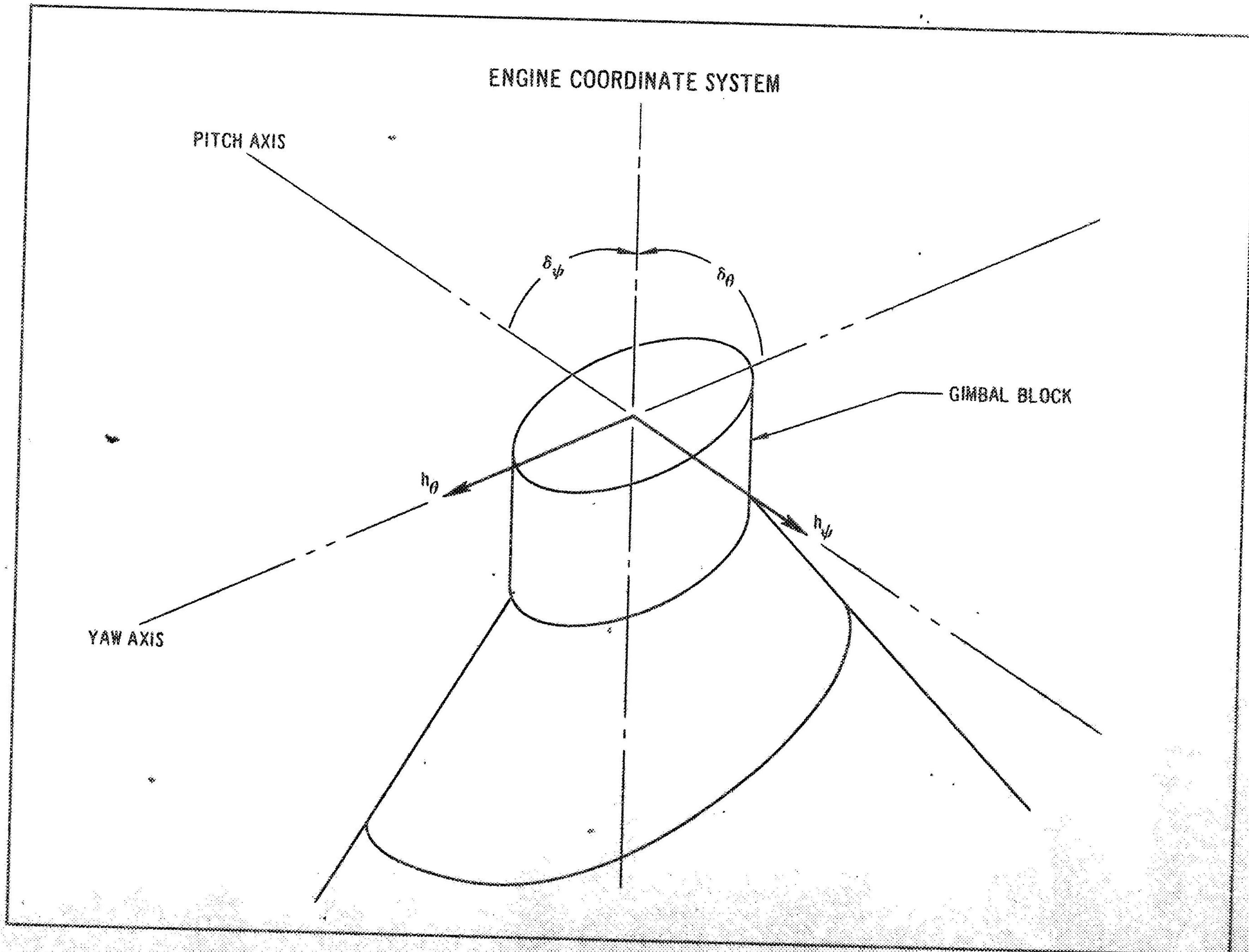

ENGINE COORDINATE SYSTEM
PITCH AXIS
δ_ψ
δ_θ
GIMBAL BLOCK
h_θ
h_ψ
YAW AXIS

FIGURE 2

$$\begin{Bmatrix} \delta_\psi \\ h_\psi \\ \delta_\Theta \\ h_\Theta \end{Bmatrix} = \begin{bmatrix} \frac{1}{\ell_\psi} & 0 & 0 & 0 & -\frac{1}{\ell_\psi} \\ 0 & 0 & 1 & 0 & 0 \\ \frac{1}{\ell_\psi} & 0 & 0 & -\frac{1}{\ell_\psi} & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{Bmatrix} \qquad (2)$$

Or

$$\{\delta_i\} = [T^{-1}] \{h_i\} \qquad (2)$$

The matrix $[T^{-1}]$ will henceforth be referred to as the "coordinate transformation matrix".

2.2.2 Force Transformation

The force matrix must also be transformed into the engine coordinate system. The forces of equation 1 are defined along the lines of deflection shown in Figure 1. Matrix equation 3 describes the thrust structure forces in terms of the engine coordinate forces and moments.

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix} = \begin{bmatrix} \frac{1}{\ell_\psi} & 0 & \frac{1}{\ell_\theta} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\ell_\theta} & 0 \\ \frac{-1}{\ell_\psi} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} M_\psi \\ P_\psi \\ M_\Theta \\ P_\Theta \end{Bmatrix} \qquad (3)$$

A check of the transformation matrix of equation 3 shows that it is the transpose of the coordinate transformation matrix.

Therefore, equation 3 may be rewritten as:

$$\{P_i\} = [T^{-1}]^* \{P'_i\} \qquad (3)$$

2.2.3 Influence Coefficients in Engine Coordinates

The equations formed thus far are repeated below.

$$\left\{h_i\right\} = \left[\phi_s\right] \left\{P_i\right\} \tag{1}$$

$$\left\{\delta_i\right\} = \left[T^{-1}\right] \left\{h_i\right\} \tag{2}$$

$$\left\{P_i\right\} = \left[T^{-1}\right] \left\{P'_i\right\} \tag{3}$$

The influence coefficient matrix can now be converted into the engine coordinate system by substituting equations 2 and 3 into equation 1.

$$\left\{\delta_i\right\} = \left[T^{-1}\right] \left[\phi_s\right] \left[T^{-1}\right]^{*} \left\{P'_1\right\} \tag{4}$$

The operation $\left[T^{-1}\right] \left[\phi_s\right] \left[T^{-1}\right]^{*}$ represents the influence coefficient matrix for the engine coordinate system. This matrix shall henceforth be denoted by $\left[\phi'_s\right]$.

For $\ell_\theta = \ell_\psi =$ 11.9 inches, the numerical value of the engine coordinate system influence coefficient matrix, $\left[\phi'_s\right]$, is:

$$\left[\phi'_s\right] = (10^{-9}) \begin{bmatrix} 18.00 & 116.59 & -4.11 & -6.22 \\ 116.59 & 4844.00 & -6.22 & -40.00 \\ -4.11 & -6.22 & 18.00 & 116.59 \\ -6.22 & -40.00 & 116.59 & 4844.00 \end{bmatrix} \tag{5}$$

3. SIMPLIFICATION OF INFLUENCE COEFFICIENTS BY REDUCTION OF COORDINATES

The system defined by the matrix of equation 4 when used for control system analysis results in excessive complication. An investigation was performed, therfore, to ascertain if the systen could be simplified without excessive loss of accuracy. One approach that could be used to get rid of unnecessary coordinates would be to use all four degrees of freedom in a stability analysis and eliminate any degrees of freedom that do not appreciably affect the results. An easier approach, which is used herein, would be to form a dynamic spring-mass system of the thrust structure and engine and retain only those degrees of freedom that comprise the low frequency modes. In this case, frequencies and mode shapes for a spring-mass system involving the four degrees of freedom from equation 4 were determined. The four resultant modes were taken to mean the answers of the "rigorous approach." Then, as many degrees of freedom were eliminated as was possible without straying too far from the rigorous approach.

3.1 Addition of Actuator and Engine Spring Constants to the System

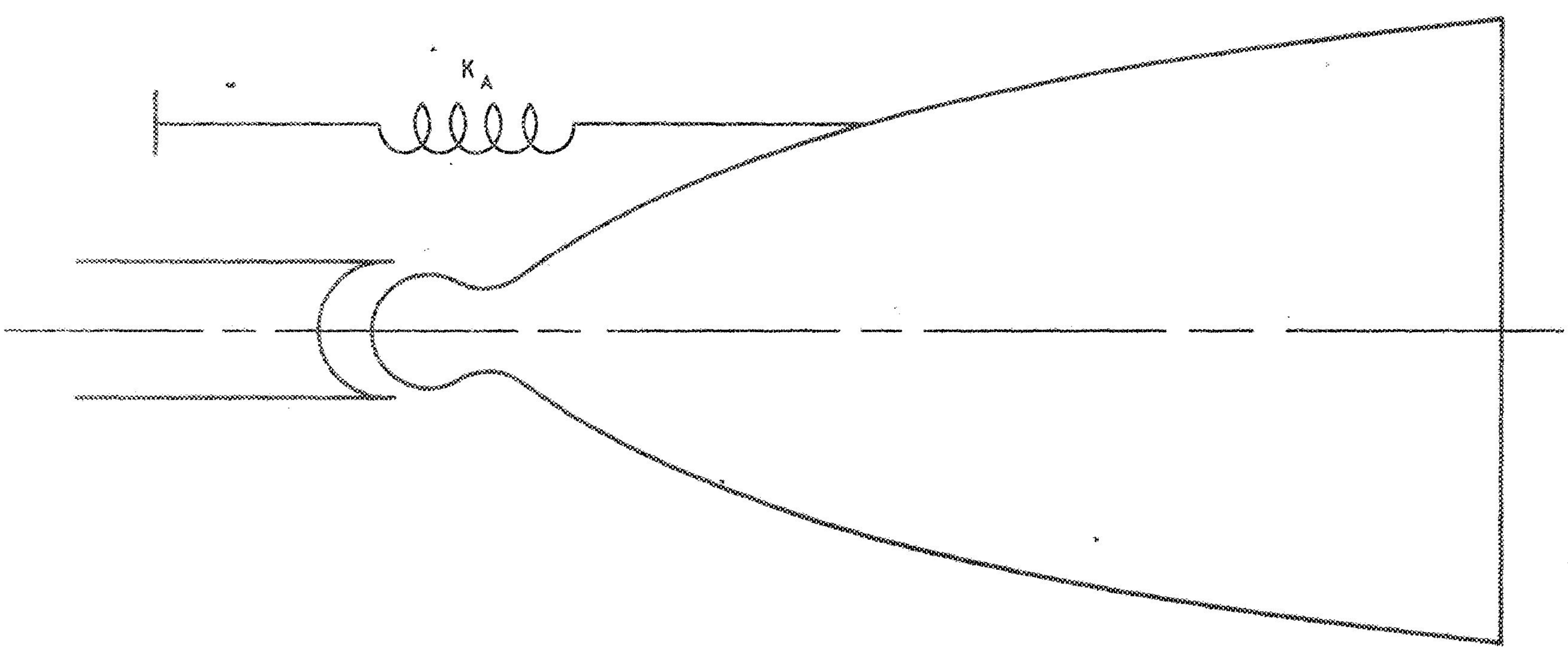
To maintain a spring-mass system, only the spring terms of the actuator were retained for this analysis. The springs of the engine and actuator add in series and can be lumped into one as shown in Figure 3. This lumped spring will hencefore be referred to as the actuator spring, Ka.

3.1.1 Actuator Spring Influence Coefficient Matrix in Thrust Structure Coordinates

This spring will only increase flexibility in the actuator attach point degrees of freedom (h_4 and h_5). The influence coefficient matrix equation for this added flexibility is then:

$$\begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/K_A & 0 \\ 0 & 0 & 0 & 0 & 1/K_A \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix} \qquad (6)$$

ACTUATOR SPRING



$$K_A = \frac{1}{\frac{1}{K_\alpha} + \frac{1}{K_E}}$$

FIGURE 3

For an estimated value of $K_A = 0.780 \times 10^6$ lb/in, equation 6 becomes:

$$\begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{Bmatrix} = (10^{-6}) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.28 & 0 \\ 0 & 0 & 0 & 0 & 1.28 \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{Bmatrix} \quad (7)$$

or

$$\{h_i\} = [\phi_A] \{P_i\}$$

3.1.2 Transformation to Engine Coordinates

The transformation from thrust structure deflections and forces to engine deflections and forces was accomplished by the same transformation procedures as in equation 4.

$$\{\delta_i\} = [T^{-1}] [\phi_A] [T^{-1}]^* \{P'_i\} \quad (8)$$

3.1.3 Actuator Spring Influence Coefficient Matrix in Engine Coordinates

The matrix operation $[T^{-1}] [\phi_A] [T^{-1}]^*$ represents the actuator influence coefficient matrix in the engine coordinate system. This matrix will henceforth be denoted by $[\phi'_A]$. The numerical value of $[\phi'_A]$ is given below in equation 9.

$$[\phi'_A] = (10^{-9}) \begin{bmatrix} 9.03 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 9.03 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

3.1.4 Combined Influence Coefficient Matrix

The total influence coefficient matrix is formed by adding equations 5 and 9.

$$\left[\phi'_T\right] = \left[\phi'_s\right] + \left[\phi'_A\right] \tag{10}$$

The numerical value of equation 10 is:

$$\left[\phi'_T\right] = (10^{-9}) \begin{bmatrix} 27.03 & 116.59 & -4.11 & -6.22 \\ 116.59 & 4844.00 & -6.22 & -40.00 \\ -4.11 & -6.22 & 27.03 & 116.59 \\ -6.22 & -40.00 & 116.59 & 4844.00 \end{bmatrix} \tag{11}$$

The matrix of equation 11 defines the influence coefficients for the thrust structure and actuator spring combined.

3.2 Four Degree of Freedom System Analysis

3.2.1 Engine Mass Matrix

The matrix of equation 11 was then used, along with the engine mass characteristics (Reference 2), to determine the vibrational modes of the system. The inertial properties of the engine are given as:

$$\begin{Bmatrix} M_\psi \\ P_\psi \\ M_\theta \\ P_\theta \end{Bmatrix} = \begin{bmatrix} 1.74\text{x}10^4 & 235.5 & 0 & 0 \\ 235.5 & 7.96 & 0 & 0 \\ 0 & 0 & 1.74\text{x}10^4 & 235.5 \\ 0 & 0 & 235.5 & 7.96 \end{bmatrix} \begin{Bmatrix} \ddot{\delta}_\psi \\ \ddot{h}_\psi \\ \ddot{\delta}_\theta \\ \ddot{h}_\theta \end{Bmatrix} \tag{12}$$

3.2.2 Mode Shapes and Frequencies

The frequencies and mode shapes for this four degree of freedom system are listed below:

Mode	Frequency (cps)	Mode Shape	
ξ_1	6.40	$\delta_\psi = 1.0$	(rad)
		$h_\psi = 5.97$	(inch)
		$\delta_\theta = -1.0$	(rad)
		$h_\theta = -5.97$	(inch)
ξ_2	7.33	1.0	
		7.49	
		1.0	
		7.49	
ξ_3	37.17	-0.0150	
		1.0	
		0.0150	
		-1.0	
ξ_4	38.33	-0.0154	
		1.0	
		-0.0154	
		1.0	

3.2.3 Integrated Mass Matrix

The Integrated Mass Matrix for the above modes is:

$$\begin{Bmatrix} P_{\xi_1} \\ P_{\xi_2} \\ P_{\xi_3} \\ P_{\xi_4} \end{Bmatrix} = (10^4) \begin{bmatrix} 4.908 & 0 & 0 & 0 \\ 0 & 4.274 & 0 & 0 \\ 0 & 0 & 4.279 & 0 \\ 0 & 0 & 0 & 4.083 \end{bmatrix} \begin{Bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_2 \\ \ddot{\xi}_3 \\ \ddot{\xi}_4 \end{Bmatrix} \tag{13}$$

3.2.4 Importance of the Four Modes Upon Control System

The first and second mode are both within the frequency spectrum of the engine servo loop. The instantaneous centers of rotation are 6 inches and 7 1/2 inches forward of the gimbal point for the two modes respectively. Thus, the modes are predominately rotation.

Conversely, the two latter modes are well out of the frequency spectrum of the control system and have centers of rotation well removed from the gimbal point -- approximately 65 inches. The former fact is sufficient reason to warrant the elimination of ξ_3 and ξ_4 from the control system analysis. However, in control analysis, descriptive coordinates such as engine rotation are more expedient to work with than the generalized coordinates made up of combined motion in the case of the modes listed above. Therefore, the descriptive coordinates (δ_ψ, h_ψ, δ_Θ and h_Θ) are the ones that must be operated upon to eliminate the generalized coordinates ξ_3 and ξ_4.

The decision as to which degrees of freedom should be eliminated is made easy by the fact that the two higher modes are predominately translation. Therefore, elimination of h_ψ and h_Θ is suggested.

3.3 Two Degree of Freedom System

3.3.1 Modified Mass and Influence Coefficient Matrices

Elimination of the translational (h) degrees of freedom results in the influence coefficient matrix contained in equation number 14.

$$\begin{Bmatrix} \delta_\psi \\ \delta_\Theta \end{Bmatrix} = (10^4) \begin{bmatrix} 27.03 & -4.11 \\ -4.11 & 27.03 \end{bmatrix} \begin{Bmatrix} M_\psi \\ M_\Theta \end{Bmatrix} \qquad (14)$$

The corresponding reduced mass matrix is presented within matrix equation 15.

$$\begin{Bmatrix} M_\psi \\ M_\Theta \end{Bmatrix} = (10^4) \begin{bmatrix} 1.74 & 0 \\ 0 & 1.74 \end{bmatrix} \begin{Bmatrix} \delta_\psi \\ \delta_\Theta \end{Bmatrix} \qquad (15)$$

3.3.2 Mode Shapes and Frequencies

The frequencies and mode shapes for this two degree of freedom system are given below.

Mode	Frequency	Mode Shape
ξ'_1	6.85 cps	$\delta_\psi = 1.0$ (rad)
		$\delta_\theta = -1.0$ (rad)
ξ'_2	7.96 cps	1.0
		1.0

3.3.3 Integrated Mass Matrix

The Integrated Mass Matrix for this reduced case is:

$$\begin{Bmatrix} P'_{\xi 1} \\ P'_{\xi 2} \end{Bmatrix} = (10^4) \begin{bmatrix} 3.48 & 0 \\ 0 & 3.48 \end{bmatrix} \begin{Bmatrix} \ddot{\xi}'_1 \\ \ddot{\xi}'_2 \end{Bmatrix} \qquad (16)$$

The 1,1 and 2,2 terms differ from the 1,1 and 2,2 terms of equation 13 by 15 per cent and 18 per cent respectively. The frequencies of the first and second modes of the rigorous approach and the latter approach disagree by 7 per cent and 8.6 per cent respectively. The resultant mode shapes in the two approaches agree exactly for the engine rotation degrees of freedom.

3.4 Determination of Engine Spring Rate

If the foregoing tolerances are acceptable, only the rotation terms of equation 4 need be included in the influence coefficient matrix. Crossing out the "h" terms of equation 5 results in the reduced influence coefficients describing the thrust structure compliance.

$$\begin{Bmatrix} \delta_\psi \\ \delta_\theta \end{Bmatrix} = (10^{-9}) \begin{bmatrix} 18 & -4.11 \\ -4.11 & 18 \end{bmatrix} \begin{Bmatrix} M_\psi \\ M_\theta \end{Bmatrix} \qquad (17)$$

Transforming the deflections from engine coordinates to actuator attach deflections:

$$\begin{Bmatrix} h_4 \\ h_5 \end{Bmatrix} = \begin{bmatrix} 11.9 & 0 \\ 0 & 11.9 \end{bmatrix} \begin{Bmatrix} \delta_\psi \\ \delta_\theta \end{Bmatrix} \tag{18}$$

or

$$\left| h_i \right| = \left[T^{-1} \right] \left| \delta_i \right|$$

Performing $\left[T^{-1}\right] \left[\phi\right] \left[T^{-1}\right]^{*}$ as before:

$$\begin{Bmatrix} h_4 \\ h_5 \end{Bmatrix} = (10^{-6}) \begin{bmatrix} 2.55 & -0.584 \\ -0.584 & 2.55 \end{bmatrix} \begin{Bmatrix} P_4 \\ P_5 \end{Bmatrix} \tag{19}$$

Therefore, the spring under the actuator is $(1/2.55) \times 10^6 = 0.392 \times 10^6$ lb/inch. The per cent cross coupling is $-0.584/2.55 = -22.9\%$.

4. MECHANICAL ANALOG

A mechanical model can be devised which will possess the same dynamic properties as the two degree of freedom system. This model is given in Figure 4.

The engine mass characteristics are represented by a mass which is attached to the gimbal block by a rigid arm. The length of this arm is equal to the length from the gimbal point to the engine center of gravity. The arms attaching the springs to the engine mass arm are equal in length to the radial distance from the spring attach points to the vehicle centerline. The flexibility of the thrust structure is simulated by the four springs thus, four conditions are effected by the model:

1. Engine mass, pitch and yaw inertia, and pitch and yaw center of gravity are simulated.
2. The engine is pinned by a ball socket and can move in any combined pitch and yaw motion.
3. Springs are provided which allow an elasticity to the actuator attach point and permit spring coupling between the two planes.
4. Symmetry is achieved by attaching the actuators 45° on either side of the platform pivot.

The model is complete as shown except for the actual values of the four springs. Spring sizing is achieved by making the influence coefficients of the mechanical analog match the influence coefficients of equation 17. Influence coefficients for the mechanical analog are obtained by imposing unit couples on the pivoted mass (engine) and calculating the angular deflections in pitch and yaw for each loading. The result of this manipulation are the influence coefficients of equation 20.

$$\begin{Bmatrix} \delta_\psi \\ \delta_\theta \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{K_\psi l_\psi^2} + \dfrac{1}{2K'_\psi l_\psi^2} & -\dfrac{1}{2K'_\psi l_\psi^2} \\ -\dfrac{1}{2K'_\psi l_\psi^2} & \dfrac{1}{K_\psi l_\psi^2} + \dfrac{1}{2K'_\psi l_\psi^2} \end{bmatrix} \begin{Bmatrix} M_\psi \\ M_\theta \end{Bmatrix} \tag{20}$$

Equation 20 is the equivalent of equation 17. The off diagonals must be equal to the off diagonal terms of equation 17.

$$-\frac{1}{2K'_{\psi}\, l_{\psi}^{2}} = -4.11 \times 10^{-9}$$

if $l_{\psi} = 11.9$ in.

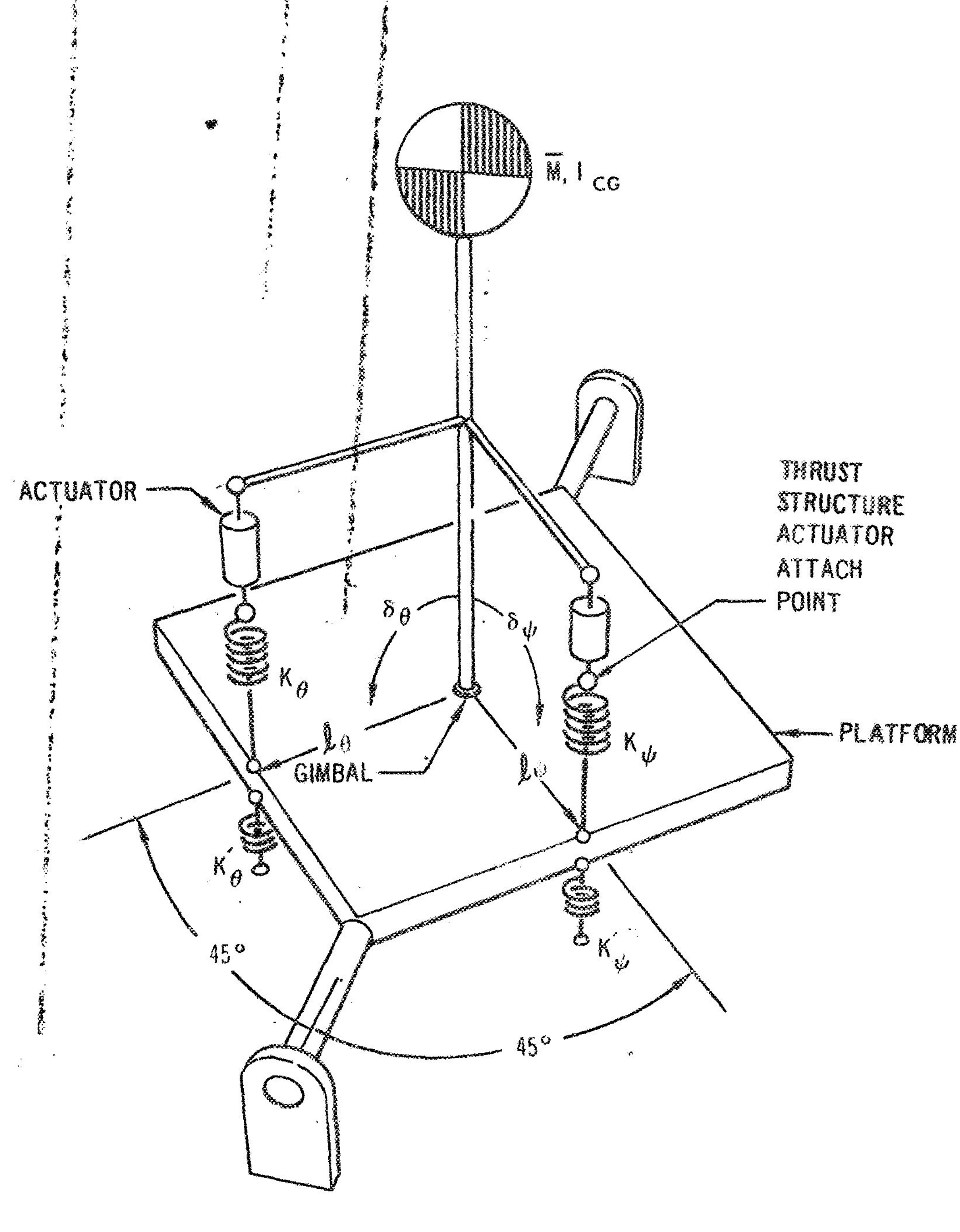
$K'_{\psi} = 0.86 \times 10^{6}$ lbs/in

The diagonals of equation 20 must be equal to the diagonals of equation 17.

$$\frac{1}{K_{\psi}\, l_{\psi}^{2}} + \frac{1}{2K'_{\psi}\, l_{\psi}^{2}} = 18 \times 10^{-9}$$

$K_{\psi} = 0.508 \times 10^{6}$ lbs/in

The inertial characteristics of the inverted pendulum would be identical to the engine.



MECHANICAL ANALOG

FIGURE 4

CONCLUSIONS

The analysis proved it is possible to decrease the degrees of freedom and still retain a tolerable accuracy. It was found that the rotational degrees of freedom dominate. A two degree of freedom system using rotation in each plane was the simplest system derived. A 7% to 9% frequency error was introduced by this reduction. Any further reduction, i.e., to one degree of freedom, would eliminate cross coupling which is fairly sizeable.

An effective spring constant for the thrust structure was determined from the two degree of freedom system. The value of this spring constant was found to be 0.392×10^6 lbs/in.

A dynamic model was then derived for the two degree of freedom system. This model is adequate for performing control system design.

REFERENCES

1. SM-46531 S-IVB Stage Thrust Structure Influence Coefficient, dated 9 December 1963

2. R-3825-1 J-2 Rocket Engine Data Manual, Rocketdyne, dated 15 August 1963